

# The Bayesian Logic of the Conjunction Fallacy: Effects of Probabilities and Frequencies in Contingency Tables

Momme von Sydow (momme.von-sydow@bio.uni-goettingen.de)

Department of Psychology, Universität Göttingen,  
Gosslersstr. 14, D-37073 Göttingen, Germany

## Abstract

In this paper a Bayesian logic of the conjunction fallacy (CF) is advocated as a normative and descriptive proposal for testing hypotheses about dyadic logical connectors. According to traditional extensional probability a violation of additivity and, in particular, a violation of  $P(A) \geq P(A \wedge B)$  or of  $P(\text{Linda is a bank teller}) \geq P(\text{Linda is a bank teller AND an active feminist})$  is a fallacy. The psychological literature has adopted this interpretation. In contrast, the proposed Bayesian model formulates qualitative as well as quantitative conditions under which such a judgment is reasonable. Qualitatively, the model is applicable to situations in which probabilities have to be taken (or are taken) not directly as extensional probabilities, but as (posterior) probabilities for alternative logical hypotheses about whole situations. If the preconditions of the model are given, Bayesian logic should be applicable to novel situations, like highly transparent tasks, even if frequency information is provided in a fully specified contingency table. Quantitatively, the model makes predictions about the effects of extensional probability patterns and resulting Bayesian probabilities and corresponding 'CFs'. Additionally, while keeping the probability patterns constant, the model predicts effects of different underlying frequencies (sample sizes). In the experiment two quantitative factors were varied using highly transparent tasks with explicit frequency information in a contingency table. Despite using these strict conditions, the results supported the predicted occurrence of 'double CFs' and the differential effects of sample size.

**Keywords:** Bayesian logic, conjunction fallacy, probability theory, hypothesis testing, noise, frequency format, logic

## The Additivity of Probability in Classical and Non-Classical Probability Theories

It is a basic truth of standard extensional probability theory that the probability of a set  $X$  can never become larger than the probability of a set  $Y$  if the latter has a larger extension than the former (inclusion rule). Applied to logical connectors and, particularly, to conjunctions, no conjunction ' $A \wedge B$ ' can be more probable than one of its conjuncts, ' $A$ ' or ' $B$ ', since the intersection of both is a subset of each conjunct. The conjunction is true for  $A \& B$  cases only, whereas, for instance, the conjunct ' $A$ ' is additionally true for  $A \& \text{non-}B$  cases. Correspondingly, probabilities have to satisfy the (extensional) conjunction rule:

$$P(A) \geq P(A \wedge B); P(B) \geq P(A \wedge B) \quad (1)$$

Taking the classical axioms of probability theory of Kolmogorov, this can be derived directly from his third axiom ( $\sigma$ -additivity): The probability of an event set which

is the union of  $n$  other disjoint subsets of events  $E_i$  is the sum of the probabilities of those subsets:

$$P(E_1 \cup E_2 \cup \dots \cup E_n) = \sum_i P(E_i) \quad (2)$$

There are alternative calculi of probability or belief which have abandoned or extended the axioms of probability theory. In our context, the most prominent approaches are the Dempster-Shafer theory of belief functions, Cohen's Baconian probabilities, and different formalizations of multi-valued or fuzzy logic (cf. e. g., Hájek, 2001, Hayek, 2003). The belief functions postulated by Dempster-Shafer theory are non-additive, but they meet the requirement of Equation 1. Likewise, the Baconian probability of a conjunction violates Equation 2 – it is equal to the minimum of the probabilities of their conjuncts, but does not violate Equation 1. Finally, in multi-valued or fuzzy logic there are quite different  $t$ -norms (Łukasiewicz, Gödel, product), but these  $t$ -norms are all consistent with  $P(A) \geq P(A \wedge B)$ .

## The Conjunction Fallacy Debate in Psychology

When Tversky and Kahneman (1983) initiated a broad psychological debate on the conjunction fallacy as a cornerstone of their 'heuristic and biases' program, they took the conjunctive rule (1) as a general norm of rational thought.

The conjunction debate was mostly concerned with tasks of the sort of the *Linda task*. In this task subjects read: "Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations." Subjects were asked to rank several statements about Linda to their probability, including: "Linda is a bank teller" ( $T$ ) and "Linda is a bank teller and she's active in the feminist movement" ( $T$  and  $F$ ). Tversky and Kahneman (1983) found that a majority of subjects judged ' $T$  and  $F$ ' to be more probable than ' $T$ ' and concluded that subjects committed a conjunction fallacy (CF) due to 'representativeness' of ' $F$ '.<sup>1</sup>

In the subsequent debate all aspects of the task and its interpretation became objects of closer scrutiny.

One group of objections concerned subtle linguistic or pragmatic aspects of the task which makes the interpretation of ' $T$ ' as well as of ' $T$  and  $F$ ' logically ambiguous, perhaps exculpating participants from committing a fallacy. A precondition for calling a judgment a 'conjunction fallacy' is that natural language roughly corresponds to the meant ideal connectors in the first place.

Firstly, in natural language ' $A$  and  $B$ ' is not always to be interpreted as a conjunction ' $A \wedge B$ ', with the common truth

<sup>1</sup> Conjunction tasks which have been explained by availability are not topic of this paper.

table function '1, 0, 0, 0' (for  $A \& B$ ,  $A \& \neg B$ ,  $\neg A \& B$ ,  $\neg A \& \neg B$ ). As Hertwig (1997, cf. Mellers, Hertwig & Kahneman, 2001) made clear, the sentence "we invite friends and colleagues to the party" often implies an inclusive disjunction of friends and colleagues (with a truth function '1, 1, 1, 0'), not their intersection. Hertwig suggested that the phrase 'bank tellers who are active in the feminist movement' would exclude this unintended interpretation as union, not testing true CFs. In Mellers, Hertwig and Kahneman (2001), Kahneman conceded that 'and' might have indeed been semantically ambiguous. However, in regard of Hertwig's 'who are' formulation he objected that this phrase is also inadequate, since it too strongly cued participants to interpret AND as a subset only. In their adversarial collaboration they settled on an "and are" formulation (we shall even use the stricter 'who' formulation). Their between-subject tests in a frequency format did not lead to a clear decision of their dispute (Mellers, et al. 2001).

Secondly, a proposition 'A', if presented in the context of a proposition 'A and B', needs not to be interpreted as the dyadic connector 'affirmation A' (with the truth function '1, 1, 0, 0'), but it may be interpreted as 'A but not B' ('0, 1, 0, 0'). "Linda is a bank teller and is active in the feminist movement" might itself prompt an interpretation of "Linda is a bank teller" as "Linda is a bank teller and not active in the feminist movement". Actually, Tversky and Kahneman (1983) themselves in one experimental condition aimed to remove this problem by using the phrase "Linda is a bank teller whether or not she is active in the feminist movement", but this only partially reduced the number of observed CFs. However, Hilton (1995, 260, cf. Tentori, Bonini & Osherson, 2004) noted that this formulation might still be misinterpreted as asserting that Linda is a bank teller even if she is a feminist. Moreover, Tversky and Kahneman did not simultaneously remove the other mentioned misunderstanding. Actually, Macdonald and Gilhooly (1990, cf. Tentori et al., 2004) did observe a much larger reduction with some problem alterations and the wording "Linda is a bank teller who may or may not be active in the feminist movement" (we shall use a similar formulation).

Another group of objections concerned the concepts of probability and of representativeness. Fiedler (1988) mentioned alternative understandings of the term 'probability' and experimentally significantly reduced the portion of CFs by using a frequency formulation instead (cf. Tversky & Kahneman, 1983). Gigerenzer (1991, cf. 1996) criticised Kahneman and Tversky's bias and heuristic approach and argued that the errors in probabilistic reasoning, like CFs, are in fact not violations of probabilistic theory, since from a frequentist perspective extensional probability theory is not applicable to single events. Kahneman and Tversky (1996) objected that giving up the inclusion rule for single events leads to normative agnosticism and empirically they showed CFs also to occur in a frequency format, at least in between-subject tasks. Gigerenzer (1996) defended his position and objected to a content-blind application of norms like the conjunction rule and to vague heuristics, like 'representativeness', as one-word explanations. To Gigerenzer, between-subject designs are not decisive, since

they do not prove a violation of internal inconsistency (also affecting Mellers, Hertwig, and Kahneman, 2001, findings).

Hertwig and Chase (1998) showed that besides a frequency format an estimation response mode also reduces CFs. Hertwig and Gigerenzer (1999) showed that the word 'probability' is polysemous, whereas the natural language sense of frequency 'is primarily mathematical'. Sloman, Over, Slovak and Stibel (2003) emphasised the importance of the ranking vs. rating responds mode and suggested that ranking may lead to understanding the options as alternatives. Recently, the existence of CFs was also shown by Tentori, et al. (2004) in transparent within-subject tasks both with a probability and a frequency format and also by Sides, Osherson, Bonini and Viale (2002) in betting contexts.

### **The Bayesian Logic of the Conjunction Fallacy**

Building on the result that 'probability' is polysemous (cf. Hertwig and Gigerenzer, 1999), a specific interpretation of probability is proposed that differs from standard extensional probability but is neither non-mathematical nor irrational. Bayesian logic provides probabilities about hypotheses concerning dyadic logical relations and whole situations (not particular cases, as in other Bayesian accounts: e. g. Fisk, 1998, cf. e. g. the BLOG model in machine learning). Although the proposed Bayesian logic may well be applicable more generally, the current article is confined to the discussion of the conjunction fallacy.

The advocated Bayesian logic is related to Oaksford and Chater's Bayesian optimal information gain approach (Oaksford and Chater, 2003) elaborated for the Wason selection task (for an extension to different probabilistic connectors, see von Sydow, 2006). However, the Bayesian logic advocated here is a model of hypotheses evaluation based on complex data patterns integrating over many noise levels and not one of information selection which is only concerned with single data points and one noise level.

The proposed Bayesian logic of hypothesis testing ('Bayesian logic' for short) inherits aspects of probability theory and of propositional logic of dyadic connectors. Nonetheless, it does not subscribe to the inclusion rule or, more particularly, to the conjunction rule. Based on an observed pattern of data,  $D$ , given in a  $2 \times 2$  contingency matrix, the Bayesian model specifies the posterior probability,  $P(H|D)$ , of different 'logical' hypotheses,  $H_k$ , like 'pupils from the Linda school generally become bank tellers' ( $B$ ) or they 'generally become bank tellers and feminists' ( $A$  and  $B$ ).

(1.) Similar to other kinds of multi-valued or fuzzy logics, this probabilistic Bayesian logic replaces the two values 'true' ('T' or '1') and 'false' ('F' or '0') of bivalent propositional logic by instead admitting truth values in the whole interval,  $[0, 1]$ , normally used for probabilities. On the logical side, Bayesian logic is basically still concerned with all 14 possible dyadic connectors,  $\circ_i$ , of propositional logic (like AND, OR, etc.) which may relate any atomic propositions  $A$  and  $B$ ,  $A \circ_i B$ , without tautology or contradiction.

(2.) More specifically, Bayesian logic assesses probabilities for hypotheses  $P_H(X)$  that concern patterns of probabilities or 'probability tables' (PTs), probabilistic analogues to deterministic truth tables. PTs are hypothetical

Tables 1a, b, c. Probability tables for three different connectors  $\text{O}_i$  and different uncertainty levels  $R = r$ .

| Table 1a |              |       | Table 1b |              |              | Table 1c |              |              |
|----------|--------------|-------|----------|--------------|--------------|----------|--------------|--------------|
| A AND B  | B            | Non-B | ONLY A   | B            | Non-B        | A OR B   | B            | Non-B        |
| A        | $t - (t-c)r$ | $cr$  | A        | $t - (t-c)r$ | $t - (t-c)r$ | A        | $t - (t-c)r$ | $t - (t-c)r$ |
| Non-A    | $cr$         | $cr$  | Non-A    | $cr$         | $cr$         | Non-A    | $t - (t-c)r$ | $cr$         |

Note: The probability  $c$  of convergence for maximal uncertainty is here set to .25.

constructs that can be tested against data. Dyadic Bayesian logic is confined to PTs based on tuples of four probabilities ( $P(A \wedge B) + P(A \wedge \neg B) + P(\neg A \wedge B) + P(\neg A \wedge \neg B) = 1$ ).

(3.) Logical connectors and hypotheses about probability tables are linked by the two assumptions of idealization and uncertainty. According to the *assumption of idealization* the connector is based on a deterministic relation with a basically equal probability distribution for true cases. The *assumption of uncertainty* (noise, error, risk, or randomness) assumes some general level of uncertainty  $R$  for a natural set of observations of a relation. This corresponds to the fact that we live in an uncertain world, with only probabilistic relations (objective uncertainty) or limited knowledge (subjective uncertainty). Rational models of testing hypotheses about logical relations under uncertainty are needed. Only in the borderline case of  $R = 0$  a single disconfirmatory case should falsify a hypothesis. The model asserts that uncertainty/noise is equally distributed over the PT. The actual  $R$  value  $r$  ( $0 \leq r \leq 1$ ) may be fixed by prior knowledge or can be calculated from the model itself.

Mathematically, the probability of a false case in a PT with  $R = 0$  is zero:  $P(F_{\text{vi}}) = f = 0$ . The probability of a true case,  $P(T)$ , in such a PT is weighted by the number of true cells of the connector under investigation:  $P(T_{\text{vi}} | R = 0) = 1 / N(T_{\text{vi}}) = t$  (cf. Table 1). If the error term approaches its maximum the PTs of all connectors  $\text{O}_i$  in question converge at a pattern where all cases have the same probability,  $c = .25$ . Formally, for any  $PT(\text{O}_i, r)$  the probability of a true case  $T$  (now with noise) is  $t - r(t - c)$ . Likewise, noise increases the zero probability of a false case  $F$  by  $r$  multiplied with the convergence value  $c$ . This formalization of randomness levels is coherent with the idea that of all true cases of a connector a portion  $r$  is distributed at random over all four possible cases (including other true cases). Table 1 provides examples for the PTs of the hypothesis of a conjunction 'A AND B', an affirmation 'A' and an inclusive disjunction 'A OR B'. Here the error level is modeled as a discrete variable (in steps of .10). Please note that the combined hypothesis  $H_k$  represent a connector combined with an uncertainty level ( $H_k = \text{O}_i \wedge R_j$ ). For the experiment, the prior probabilities for the hypotheses  $H_k$  are assumed to be equal.

(4.) We now calculate the probability of some observed data pattern given one hypothesis,  $P(D | H_k)$ . A data sample, ordered in a  $2 \times 2$  contingency matrix,  $D$ , consists of four frequencies,  $x_1, x_2, x_3, x_4$  (with  $\sum x_i = n$ ). The multinomial distribution gives the discrete probability distribution  $P(x_1, x_2, x_3, x_4 | n, p_1, p_2, p_3, p_4) = P(x_i | n, p_m)$  of obtaining a particular pattern of the four disjoint outcomes,  $x_1, x_2, x_3, x_4$ , in a total sample of  $n$  independent trials given a hypothesis with the respective probabilities  $p_1, p_2, p_3, p_4$  (with  $0 \leq p_m \leq 1, \sum p_m = 1$ ). It has the following probability mass function:

$$P(x_i | n, p_m) = \binom{n}{x_1, x_2, x_3, x_4} p_1^{x_1} p_2^{x_2} p_3^{x_3} p_4^{x_4} \quad (3)$$

(5.) In order to calculate the posterior probabilities of each combination of connector and uncertainty level,  $H_k$ , given the observed pattern of data,  $D$ , Bayes' theorem is used:

$$P(H_k | D) = \frac{P(D | H_k)P(H_k)}{P(D)} \quad (4)$$

The normalizing probability  $P(D)$  of the data  $D$  under all hypotheses  $H_k$  (connector-uncertainty combinations,  $\text{O}_i \times R_j$ ) is calculated by:

$$P(D) = \sum_{k=1} P(D | H_k)P(H_k) \quad (5)$$

As simple measure of information gain, one can calculate the impact of the data on the probability of each hypotheses:

$$P_{\text{Diff}}(H_k, D) = P(H_k | D) - P(H_k) \quad (6)$$

Since we are here concerned with *logical* hypotheses ( $H_i = \text{O}_i$ ) without a specified error level, and since we calculated the posterior error levels from the data, we have to formulate an integration rule to determine the global probability of the logical hypotheses in question. Here for each  $H_i$  the sum of the posterior probabilities over all error levels  $r_j$  is calculated, resulting in a probability mass function.<sup>2</sup>

$$P(H_i | D) = \sum_j P(H_{i,j} | D) \quad (7)$$

## Predictions of Bayesian Logic

Bayesian logic provides a suitable alternative to traditional extensional probability, not replacing it, but supplementing it. Bayesian logic (itself based on extensional probabilities) is meant as a rational formalization of probabilities for alternative hypotheses of connectors corresponding to the whole of a probabilistic truth table ('hypotheses probability' or  $P_H(X)$  for short). In contrast, extensional probability theory provides us with probabilities of those subsets of a PT which are specified by true extensions of the corresponding logical connectors ('extensional probability' or  $P_E(X)$  for short). Here we are concerned with  $P_H(X)$  in the context of the conjunction fallacy, considering two kinds of predictions, although only the latter one is varied in our study:

<sup>2</sup> Alternatively one may weight  $R$  values by their reciprocal,  $1/r$ . In the experiments conducted here, data sets have been used, for which both models lead to the same predictions.

(1.) *Qualitative predictions.* From the preconditions of the model one can derive constraints for a situation in which the outlined model should be normatively and descriptively applicable: In the reported experiment we are going to construct situations in which it is plausible to understand probabilities as *alternative hypotheses* (cf. Hertwig & Chase, 1998, Sloman et al., 2003) about connectors, each relating to a whole situation (not to a subset). If such preconditions of the model are met, it is claimed that one can achieve a substantial portion of ‘conjunction fallacies’ even with extremely transparent tasks, salient frequency information, and excluded misunderstandings (concerning ‘A and B’ and ‘A’). Here I will use explicit contingency tables (going beyond the experiments of Sloman et al., 2003, and Tentori et al., 2002). Nonetheless, Bayesian logics (unlike e. g. Gigerenzer, 1996) predicts CFs.

(2.) *Quantitative predictions.* Here we are particularly concerned with two novel predictions of the model, both distinguishing  $P_H(X)$  from  $P_E(X)$ . According to Bayesian logic there are quantitative conditions, in which a hypothesis with a narrower extension may have a higher hypothesis probability,  $P_H(X)$ , than a one with a broader extension.

Firstly, one prediction is concerned with the probability pattern given by the data if the sample sizes are large, as in Example 1 and 2 of Figure 1. Here, whether a CF should occur or not, should depend on the probability pattern of the data. Example 1 provides a data pattern under which a novel double CF effect (with  $P_H(A) \cong P_H(B) < P_H(A \wedge B)$ ) is predicted. In contrast, Kahneman and Tversky were concerned only with single CFs, if feature A is more ‘representative’ than feature B (the prediction  $P(B) < P(A \wedge B)$ ). However, for Example 1 a double focus effect

**Figure 1:** Graphs of information gain for three hypotheses ‘A & B’, ‘A’, and ‘B’ given the observed frequencies, for each uncertainty level ( $P(H_k|D)-P(H_k)$ ,  $r = .1$  to 1.0, left) or summing up the weighted noise levels ( $P(H_i|D)-P(H_i)$ , right).

Figure 1a:  
Example 1,  
Graphs for  
frequency pattern  
'26, 13, 14, 12'.

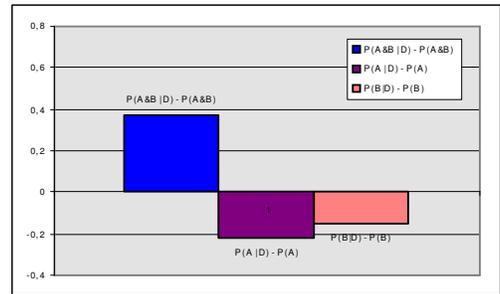
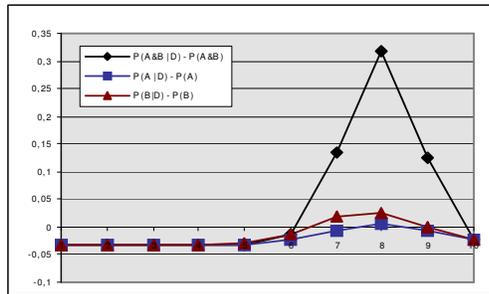


Figure 1b:  
Example 2,  
Graphs for  
frequency pattern  
'26, 25, 14, 12'.

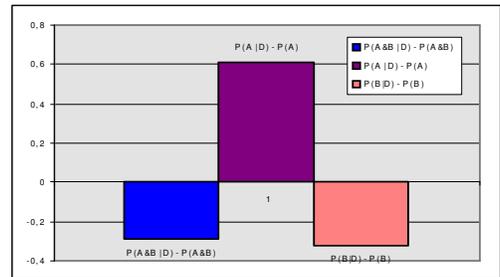
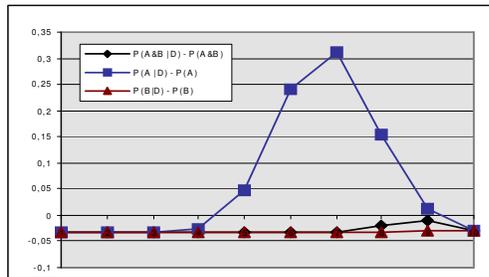


Figure 1c:  
Example 3,  
Graphs for  
frequency pattern  
'2, 1, 1, 1'.

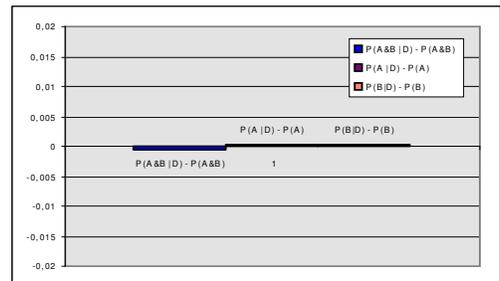
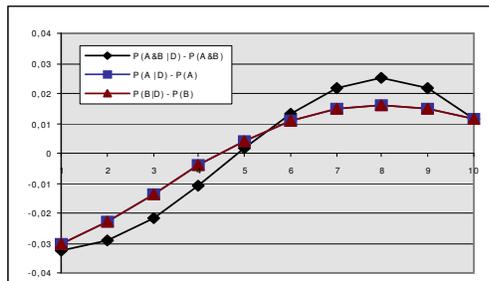
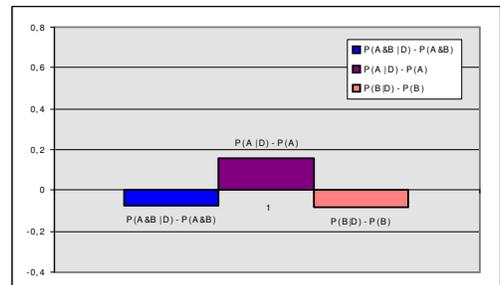
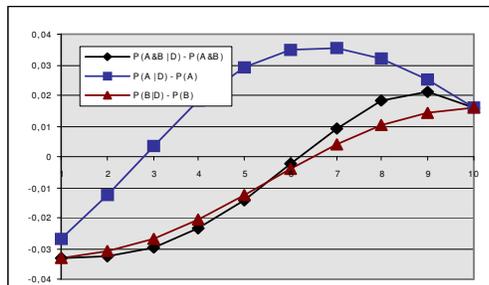


Figure 1d:  
Example 4,  
Graphs for  
frequency pattern  
'2, 2, 1, 1'.



and for Example 2 no or only a weak single CF effect is predicted (the latter point is not tested here). The Bayesian logic of hypotheses concerning whole PTs obviously differs from Fisk's (1998) Bayesian model related to extensional subsets only, which never allows for  $P(A \wedge B | D) > P(A | D)$ .

Secondly, when keeping the probabilities  $P_E(X)$  constant the model predicts no change in the discussed Bayesian probabilities,  $P_H(X)$ , provided there is a substantial sample size. Whereas extensional probabilities should remain unaffected by sample size and traditional extensional sampling statistics would differ for large and medium sample sizes, Bayesian probabilities should change in a specific way if the sample size is very low. Bayesian models integrate aspects of reliability into the probability measure. The Examples 3 and 4 show that despite constant extensional probabilities the predicted Bayesian patterns get less pronounced, but the Bayesian pattern of 'A AND B' in Example 3 is clearly more affected than the pattern 'ONLY A' in Example 4.

### Experiment

The reported experiment tests the mentioned qualitative and quantitative predictions of the advocated Bayesian logics.

Qualitatively, extremely transparent ranking tasks were used, where all hypotheses,  $A \& B$ ,  $A$  and  $B$ , but no filler hypotheses were formulated. Simultaneously, the aim was to exclude the misunderstandings concerning  $P(A \& B)$  and  $P(A)$  (cf. Tentori et al. 2004). The task was conducted as within-subjects task. One objective was to test whether CFs could even be elicited under conditions where frequency information is explicitly provided in a contingency table.

Quantitatively, the frequencies of all logical cases were shown and varied according to Table 2, investigating both the effects of the probability patterns (AND versus A conditions) and of sample size (small versus large sample size).<sup>3</sup>

Table 2: The Observed Frequencies of Different Cases in Different Schools/Conditions (Linda, Maria, Sara, Nina)

|          | $A \wedge B$ | $A \wedge \neg B$ | $\neg A \wedge B$ | $\neg A \wedge \neg B$ |
|----------|--------------|-------------------|-------------------|------------------------|
| High AND | 102          | 51                | 52                | 50                     |
| High A   | 102          | 100               | 50                | 52                     |
| Low AND  | 2            | 1                 | 1                 | 1                      |
| Low A    | 2            | 2                 | 1                 | 1                      |

**Method** The 98 participants were told in the instructions that they had to find out which 'hypotheses' about schools are most probable and closest to truth. In order to fulfill the preconditions of the model (alternative hypotheses interpretation and whole PT interpretation) the task concerned *different* schools, such as a Linda school. Each hypothesis concerned a school as a whole. A 2x2 contingency table with information about a sample of pupils was given, using side labels like "bank tellers", "no bank tellers" etc. (Table 2).

Each participant investigated two schools in which the observed patterns differed both in probability and sample size. Subjects were asked to tick for each school the option, which

<sup>3</sup> The predictions are almost identical to those in Figure 1. The different examples led to broader distributions and were used to visualize that there are probability distributions over error levels.

(s)he regards to be most probable. Subjects were asked to make this judgment intuitively. The hypotheses read:

*A hypothesis:* "Today, the girls in the Linda [Maria, etc.] school are generally *bank tellers*, whether they are feminists or not." ("[...] sind heute in der Regel Bankangestellte, egal ob sie aktive Feministinnen sind oder nicht.")

*B hypothesis:* "[...] are generally *active in the feminist movement*, whether they are bank tellers or not."

*AND hypothesis:* "[...] are generally *bank tellers who are active feminists*" ("*Bankangestellte, die*", cf. Introduction).

*No hypothesis ('?')*: "Based on the data no single hypothesis is really better supported than the other hypotheses."

### Results

Table 3: Percentage and Number of Choosing Hypotheses as Being most Probable

|          | A       | B      | AND     | ?       | n  |
|----------|---------|--------|---------|---------|----|
| High AND | 17 % 8  | 17 % 8 | 42 % 20 | 25 % 12 | 48 |
| High A   | 67 % 32 | 6 % 3  | 15 % 7  | 13 % 6  | 48 |
| Low AND  | 10 % 5  | 6 % 3  | 19 % 9  | 65 % 31 | 48 |
| Low A    | 52 % 25 | 4 % 2  | 6 % 3   | 38 % 18 | 48 |

*Note:* The predicted cells are darkened.

For each condition, Table 3 summarizes the number and percentage of participants choosing a particular hypothesis as the most probable one. As predicted, the portion of 'AND' choices was significantly larger in the 'high AND' condition than in the 'high A' condition ( $\chi^2(1, n = 96) = 8.71; p < .01$ ). Apart from the '?' answers, 56 % of the participants in the 'high AND' condition committed a 'double CF' (single CFs were not tested). Likewise, the portion of 'A' choices was lower in the 'high A' condition than in the 'high AND' condition,  $\chi^2(1, n = 96) = 24.69; p < .001$ . Moreover, it was confirmed that the AND selections were significantly reduced in the 'low AND' relative to the 'high AND' condition ( $\chi^2(1, n = 96) = 5.98, p_{\text{one-tailed}} < .01$ ). In contrast, it was likewise corroborated that the A selections were *not* significantly reduced in the 'low A' relative to the 'high A' condition ( $\chi^2(1, n = 96) = 2.12; p = .15$ ) and there were more A choices in the 'low A' than in the 'low AND' condition ( $\chi^2(1, n = 96) = 19.39; p < .001$ ). Finally, it was shown that there were more '?' choices in the two high frequency conditions than in the two corresponding low frequency conditions ( $\chi^2(1, n = 96) = 15.21; p < .001; \chi^2(1, n = 96) = 8.00; p < .01$ ) and that there were more such choices in the 'low AND' condition than in the 'low A' condition ( $\chi^2(1, n = 96) = 7.04; p < .01$ ). A slightly different replication of the study led to similar results.

### Discussion

The results of the experiment were predicted by Bayesian logic. The effects of probability patterns as well as the differential effects of sample size were corroborated.

The 'high AND' condition elicited estimations corresponding to  $P(A \wedge B) > P(A)$  and  $P(A \wedge B) > P(B)$ . The expected double CFs were confirmed for the first time in situations with extremely transparent tasks, clear set inclusion and explicit frequency information in a contingency table. According to extensional probability the correct answer in the 'high AND' condition would have been 'B', but the 'B'

choices occurred only as often as the 'A' choices. It is particularly problematic for the frequentist position (e. g. Gigerenzer, 1996) that CFs occur even if a full contingency table is provided.

Furthermore, the specific predictions regarding set size were also confirmed. High and low sample conditions had roughly the same extensional probabilities, but the posterior probabilities of the tested hypotheses,  $P_H(H_i|D)$ , differed and were actually judged differently. The number of answers without a clear preference rose. More importantly, as predicted, in the 'Low AND' condition the 'A AND B' selection did not remain the modal answer, whereas in the 'Low A' condition the 'A' selection was reduced but remained predominant. This supports Bayesian logic.

One may perhaps think that the results are post hoc also explainable by another formal model in the CF debate. As mentioned before, Fisk's (1996) Bayesian model, based on extensional subsets, does not allow for any rational CFs. Tversky and Koehler's (1994) support theory cannot explain the results, since there should be an equal unpacking in all conditions. Replacing probability by reverse probability is another plausible candidate (cf. Fisk, 1996; Hertwig & Chase, 1998; Sides et al. 2002, p. 191-192) but it is difficult to see how one may apply this interesting idea to our frequency table tasks. In traditional Linda tasks inverse probability may indeed partially exculpate participants since  $P(D|A \& B) > P(D|A)$  appears reasonable, because "Linda is more likely to be single, outspoken, and so on, on the assumption that she is a feminist bank teller than on the mere assumption that she is a bank teller" (Sides, et al., 2002, 192). Nonetheless, in our current study there are no varying characteristics of Linda, but only different frequencies. Any single-cue explanation (Hertwig & Chase, 1998) is excluded, since, given that this effect appears to refer to an interaction of two cues, it cannot account for the novel double focus effects. But how to formalize inverse probability here? Using the formalization of Bayesian logic would go beyond previous models. Interpretations which lead to  $P(D|H) = 1$  or collapse with extensional probability ( $P(D|A \& B) = P_E(A \& B)$ ) have to be excluded. Interpreting  $P(A)$  as average probability ( $P(A)_{AV} = P_E(A \wedge B) + P_E(A \wedge \sim B) / 2$ ) allows for violation of the conjunction rule, but this measure would here falsely predict CFs for the 'high A' condition as well. This would not be improved, if we assumed different prior probabilities for  $A \wedge B$  and  $A$ : Without frequency information the original probability  $P(\text{Bank teller} \wedge \text{Feminist})$  would be judged to be lower than the probability for its compounds. Hence, the difference between the original estimation and the resulting estimation given the frequencies would even clearly lead to falsely predict CFs in the 'low A' condition as well.

In conclusion, the results corroborate Bayesian logic and have not been predicted by any other theory of the CF.

It remains an open question, whether the developed Bayesian logic has to be understood as a more precise formalization of the vague heuristic 'representativeness', or whether it constitutes a third concept beside extensional probability and representativeness (cf. Gigerenzer, 1996, Gigerenzer et al. 1999). In any case, Bayesian logic shows that 'CFs' may (partly) not be 'fallacies' at all, even if we are concerned with the evaluation of contingency tables.

## Acknowledgements

I thank Björn Meder and three reviewers for helpful comments. Thanks to my research group for inspiration. I also want to thank the *Deutsche Forschungsgemeinschaft* for a grant 'Bayeslogik' (Sy 111/1-1) that made this work possible.

## References

- Fiedler, K. (1988). The dependence of the conjunction fallacy on subtle linguistic factors. *Psychological Research*, 50, 123-129.
- Fisk, J. E. (1996). The Conjunction Effect: Fallacy or Bayesian Inference? *Organizational Behavior and Human Decision Processes*, 67, 76-90.
- Gigerenzer, G. (1996). On Narrow Norms and Vague Heuristics: A Reply to Kahneman and Tversky (1996). *Psychological Review*, 103, 592-596.
- Hájek, A. (2001). Probability, Logic, and Probability Logic. In L. Goble (Ed.), *The Blackwell Companion to Logic* (362-384), Oxford: Blackwell.
- Hájek, P. (2002). Fuzzy Logic. In Zalta, E. N. (Ed.), *The Stanford Encyclopedia of Philosophy*. URL: <http://plato.stanford.edu/entries/logic-fuzzy/> (3<sup>rd</sup> Sep.).
- Hertwig, R., & Chase, V. M. (1998). Many reasons or just one: How response mode affects reasoning in the conjunction problem. *Thinking and Reasoning*, 4, 319-352.
- Hertwig, R., & Gigerenzer, G. (1999). The 'Conjunction Fallacy' Revisited. *Journal of Behavioral Decision Making*, 12, 275-305.
- Kahneman, D., & Tversky, A. (1996). On the reality of cognitive illusions. *Psychological Review*, 103, 582-591.
- Mellers, B. A., Hertwig, R., & Kahneman, D. (2001). Do frequency representations eliminate conjunction effects? *Psychological Science*, 12, 269-275.
- Moutier, S., & Houdé, O. (2003). Judgement under uncertainty and conjunction fallacy inhibition training. *Thinking and Reasoning*, 9, 185-201.
- Oaksford, M., & Chater, N. (2003). Optimal data selection: Revision, review, and reevaluation. *Psychological Bulletin & Review*, 10, 289-318.
- Sides, A., Osherson, D., Bonini N., & Viale, R. (2002). On the reality of the conjunction fallacy. *Memory and Cognition*, 30, 191-198.
- Slooman, St. A., Over, D., Slovak, L., & Stibel, J. M. (2003). Frequency illusions. *Organizational Behavior and Human Processes*, 91, 296-309.
- von Sydow, M. (2006). Towards a flexible Bayesian and deontic logic of testing descriptive and prescriptive rules: Explaining Content Effects in the Wason Selection Task. Doctoral dissertation, Universität Göttingen.
- Tentori, K., Bonini, N., & Osherson, D. (2004). The conjunction fallacy: a misunderstanding about conjunction? *Cognitive Science*, 28, 467-477.
- Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. *Psychological Review*, 90, 293-315.
- Tversky, A., & Koehler, D. J. (1994). Support theory: A nonextensional representation of subjective probability. *Psychological Review*, 101, 547-567.