

The Bayesian Logic of Conjunction Fallacies: Probability Rating Tasks and Pattern-Sensitivity

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Abstract

A pattern-sensitive Bayesian logic (BL) is proposed as a normative and descriptive account for testing hypotheses and explaining conjunction fallacies (CF) with frequency information. The conjunction rule, used for instance in $P(\text{Linda is a bank teller}) \geq P(\text{Linda is a bank teller AND an active feminist})$, has been claimed to be one of the most basic universal truths of probability theory. In contrast, Bayesian logic formulates qualitative as well as quantitative conditions under which violations of this rule are reasonable. Bayesian logic is not concerned with subsets, but with the overall probability that observations of features are produced in a probabilistic way by underlying logical generative structures. Here it is shown empirically that subjects commit CFs and even double CFs when simultaneously concerned with transparent tasks, frequency information, and rating tasks. It is tested whether probability judgments are pattern-sensitive in a way not predicted by alternative theories. The results provide support for BL and can neither be explained by a frequentist account nor by alternative theories of the CF.

Keywords: Probability judgments; conjunction fallacy; logic.

Introduction

The Linda task has become a hallmark of the psychological rationality debate, associated with the claim that human beings are not able to adhere even to the perhaps simplest qualitative law of probability theory (cf. e.g., Tversky & Kahneman, 1983; Kahneman & Tversky, 1996; Kahneman & Frederick, 2002).

The classic Linda task reads: “Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.” Subjects had to rank statements to their probability, including several filler items and: “Linda is a bank teller” (*B*) and “Linda is a bank teller and she’s active in the feminist movement” (*B* and *F*). Tversky and Kahneman (1983) found that most participants estimated $P(B \text{ and } F) > P(B)$. However, according to extensional probability theory no conjunction ‘ $B \wedge F$ ’ can be more probable than one of its components, ‘*B*’ or ‘*F*’, since the intersection of the components is a subset of each of them. Even according to non-standard accounts of probability, like the Dempster-Shafer theory of belief functions, Cohen’s Baconian probabilities, or fuzzy logic, which partly gave up the strict additivity of probabilities, the conjunction rule is valid. Correspondingly, Tversky & Kahneman (1983) concluded that subjects committed a ‘conjunction fallacy’ (CF), based on the use of the representativeness heuristics. The representativeness heuristics consists of a first step of evoking the representation of a prototype (here Linda is assumed to be a feminist) and a second step of judging the

similarity of Linda with that prototypical exemplar (Kahneman & Frederick, 2002, 2005). However, representativeness has been criticized as being defined in a way that is too vague, undefined, and unspecified to count as a proper explanation; instead it would only provide explanations by redescription (Wolford, 1991; Gigerenzer, 1996, cf. 1991, 1998, 2000).

Debate on the Conjunction Fallacy

The dispute on the CF gave rise to a multitude of empirical studies and theoretical considerations. It was discussed whether participants could be exculpated from committing a fallacy by analyzing every minutiae of the Linda task.

First, researchers have argued that the apparent ‘fallacy’ may be due to misunderstood logical terms, and that the interpretation of ‘*B*’ (bank teller) and of ‘*B* and *F*’ (bank teller and feminist) may be logically ambiguous (e.g., Hilton, 1995; Mellers, Hertwig, & Kahneman, 2001; cf. Sides, Osherson, Bonini, & Viale, 2002; Bonini, Tentori, & Osherson, 2004). A statement or hypothesis ‘*B*’, if presented together with a statement ‘*B* and *F*’, may be interpreted as ‘*B* but not *F*’. Actually, in one study, Tversky and Kahneman (1983) themselves already aimed to remove this problem using the phrase “Linda is a bank teller whether or not she is active in the feminist movement”, but this only partially reduced the number of observed CFs. In any case, we will use this stricter formulation. Moreover, in natural language ‘*B* and *F*’ needs not to be interpreted as a conjunction ‘ $B \wedge F$ ’, but in some contexts it may be interpreted as a disjunction ‘ $B \vee F$ ’. If one invites friends ‘and’ colleagues to a party, this does not only refer to the intersection, to those who are both friends and colleagues at the same time (Hertwig, 1997, cf. Mellers et al., 2001). One may solve this problem by using ‘persons who are colleagues and at the same time friends’. Here we use this formulation and control for any misunderstanding by varying frequencies.

Secondly, it was argued that CFs may be due to the use of single-event probabilities instead of providing frequency information. It was found that a frequency format concerned with 100 hypothetical persons fitting Linda’s description significantly reduced the portion of CFs (Tversky & Kahneman, 1983; Fiedler, 1988; Hertwig & Gigerenzer, 1999). Though CFs occurred in more complex tasks with explicit frequency information (Lagnado & Shanks, 2002) or under controlled conditions (Sloman, Over, Slovak, & Stibel, 2003), most authors conceded that a frequency format in simple tasks has some facilitating effect (Lagnado & Shanks, 2002; see even Frederick & Kahneman, 2002, 2005). Based on such findings Gigerenzer (1991, 1996, 2000) argued that CFs in the Linda task are only violations of *some* subjective theories of probability (including a

Bayesian approach), but no violations of the major frequentist conception of probability. The Linda task is only concerned with single events (Linda becomes bank teller, or she becomes a bank teller and a feminist). CFs and other ‘frequency illusions’ should ‘largely disappear’, if tasks were presented in a natural frequency format (cf. Cosmides & Tooby, 1996; Zhu & Gigerenzer, 2006), showing the final tally of the natural sampling process. It was even claimed that humans are adapted to frequencies and not to (unobservable) ‘probabilities’ of single events (Cosmides & Tooby, 1996; Gigerenzer, 1998, p. 12). In our studies we will use simple transparent tasks and frequency information, not only using 100 hypothetical Lindas, but explicit frequency information also on the conjunction itself.

Thirdly, it has also been shown that a rating response mode instead of a ranking response mode enhances the extensional performance in CF tasks (Hertwig & Chase, 1998; Hertwig & Gigerenzer, 1999; Sloman et al., 2003). Here we are going to use a rating response mode, but BL will nonetheless predict CFs.

Fourthly, the transparency of the nested-set relation between a conjunction and its components is another plausible cue that seems to facilitate an extensional solution of the task (cf. Johnson-Laird et al., 1999). Sloman, Over, Slovak, and Stibel (2003) have even provided evidence for the suggestions that logical set inclusion may explain the frequency effect. Hence, in our studies on CFs, we use clear set inclusions by using clear formulations and an explicit contingency table.

Finally, the term probability itself has been considered to be polysemous (Hertwig & Gigerenzer, 1998; cf. Fiedler, 1988; Wolford, 1991; Fisk, 1996; Sides et al., 2002). Gigerenzer (1991, 1996) objected to a content-blind application of norms like the conjunction. BL formalises an intuitive pattern-sensitive notion of probability. There are also other semi-rational mathematical theories of the CF. (cf. Wolford, 1991; Fisk, 1996; Hertwig & Chase, 1998; Lagnado & Shanks, 2002; Sides et al., 2002). However, these theories make different predictions in our study.

The Bayesian Logic of the Conjunction Fallacy

On the computational level, BL provides an alternative model to extensional probability that is neither non-mathematical nor irrational (cf. v. Sydow, 2007). BL provides probabilities about hypotheses concerned with logical relations and whole situations (not only with particular subsets, as suggested in previous Bayesian models of the CF, cf. Fisk, 1996). Although BL makes use of extensional probability theory, on the emergent level of description relevant here, it does not subscribe to extensionality and the conjunction rule. BL calculates pattern-sensitive posterior

probabilities of logical hypotheses, given some data pattern D in a 2×2 contingency matrix. The posterior probabilities $P(H_i | D)$ of alternative ‘logical’ hypotheses, H_i , like ‘pupils from the Linda school generally become bank tellers’ (A) or ‘become bank tellers and feminists’ (A and B) are specified using probabilistic probability tables and different noise levels, whose probabilities are also calculated from the data. BL is concerned with the probabilities of these tables of probabilities. The resulting hypothesis probability, $P_H(X)$, is not concerned with subsets ($P_E(X)$), but with patterns characterizing whole situations.

(1.) The main input of BL are natural frequencies which may be directly observable or retrieved from memory. Think of a sample of graduates from a particular school, for instance the Linda school. The frequency of two properties A and B (here bank teller and feminist) can be depicted in 2×2 contingency table resulting from N independent observations: $x_a + x_b + x_c + x_d = N$. Additionally, BL assumes prior probabilities of the hypotheses (combining a connector, A or B , and a noise level). For our laboratory settings we assumed an uninformative flat prior distribution.

(2.) Like other kinds of multi-valued or fuzzy logics, BL needs to combine aspects of probability theory and propositional logic. BL postulates probability tables (PTs) as probabilistic analogues to deterministic truth tables, also specifying specific noise levels. Note, that PTs are ‘Kantian’ entities based on *a priori* assumptions of the model. PTs are necessary preconditions to get this kind of inductive logic going. Based on the data, BL then calculates second order probabilities for each PT, $P(A \text{ or } B, \text{noise level} | D)$. We are here only concerned with dyadic logical relations. For constructing PTs two assumptions are made.

First, the *assumption of idealization* asserts equal probability distributions for all true cases of a hypothetical ideal connector (cf. Johnson-Laird et al., 1999). In case of deterministic logical relationships (no noise, $R = 0$), the ideal probability of a cell of a PT that is logically ‘true’ according to a given connector, is one divided by the number of true cells given by this connector: $P_{PT}(T | R = 0) = 1 / N(T_i) = t_i$. This turns deterministic truth tables into (deterministic) probability tables (cf. Table 1).

Secondly, the *assumption of uncertainty* asserts a general overall level of noise for a logical PT. A formalization of noise (probabilisticity) is needed to develop the ideas of probabilistic logical relations. $R = r$ specifies for each PT the degree to which the probabilities diverge from a deterministic connector, with r being the portion of the PT’s generative probability that is distributed equally on all cells. A particular noise value has a probability $P(R = r)$ (with $0 \leq r \leq 1$), which may be fixed by prior knowledge. However, we here calculate the posterior probability of each noise-connector combination from the data itself, generally assuming flat priors. A PT with $R = 0$, just as a truth table,

Table 1: Probability Tables for Three Connectors \circ_i , a uniform $c = .25$, and Uncertainty Levels ($R_j = r$), cf. v. Sydow, 2007.

A AND B	B	Non-B
A	$1 - (.75)r$	$.25r$
Non-A	$.25r$	$.25r$

A	B	Non-B
A	$.5 - (.25)r$	$.5 - (.25)r$
Non-A	$.25r$	$.25r$

B	B	Non-B
A	$.5 - (.25)r$	$.25r$
Non-A	$.5 - (.25)r$	$.25r$

can be discarded by a single disconfirmatory observation (a falsification). In contrast, for PTs, whose error term approach their maximum ($R = 1$), the cell probabilities converge at a pattern that (here) have the same ‘convergence probabilities’, $c = .25$ (cf. Table 1). More generally the assumed probability of a false cell in a $PT_{l,j}$ is formalized by $P_{PT}(F) = r * c$ (with $0 \leq P_{PT}(F) \leq c$) and that of a true case, T , by: $P_{PT}(T) = t_l - r (t_l - c)$, with $c \leq P_{PT}(T) \leq t$. For more arbitrary sampling processes specifying the portion of sampled entities in before, a factor s can be added to each cell of a PT representing these proportions (normalizing the resulting cell values by their overall sum).

Each hypothetical connector is combined with all hypothetical noise levels (here r only varied in steps of .10). This formalization generally specifies a two-dimensional field of PTs, ordered by connector and noise level. We here write H_k for PTs combining a connector with an uncertainty level ($H_k = PT(O_l \wedge R_l)$).

(3.) Now we can calculate the probability of observed data patterns given a combined connector-noise hypothesis, $P(D | H_k)$. The multinomial distribution provides the discrete probability distribution $P(x_a, x_b, x_c, x_d | p_a, p_b, p_c, p_d) = P(D | PT_k) = P(D | H_k)$ of obtaining a particular pattern of the disjoint outcomes, in a total sample of N independent trials, given a hypothesis (a PT) with the respective probabilities p_a, p_b, p_c, p_d (with $0 \leq p_m \leq 1, \sum p_m = 1$). For a given data and H_k (all PTs specified by the model) $P_H(D | H_k)$ is calculated.

$$P_H(D | H_k) = \binom{N}{x_a x_b x_c x_d} p_a^{x_a} p_b^{x_b} p_c^{x_c} p_d^{x_d} \quad (1)$$

(4.) Subsequently, Bayes’ theorem is used in order to calculate the posterior probabilities of each connector-uncertainty combination, H_k , given the data pattern, D :

$$P_H(H_k | D) = \frac{P(D | H_k)P(H_k)}{P(D)} \quad (2)$$

$$P(D) = \sum_{k=1} P(D | H_k)P(H_k) \quad (3)$$

This allows the calculation of the two-dimensional field of posterior probabilities of connector-noise hypotheses.

(5.) In order to calculate the probability of a *logical* hypothesis ($H_l = A \circ_l B$), we sum up the posterior probabilities of all corresponding PTs over all error levels r_j :

$$P_H(H_l | D) = \sum_j P_H(H_{l,j} | D) \quad (4)$$

(6.) If the preconditions of the model are met, and one is concerned with alternative logical hypothesis and whole situations (rather than subsets), BL makes a number of qualitative and quantitative predictions (cf. v. Sydow, 2007; v. Sydow, in prep.). The predicted pattern probabilities $P_H(A \circ B)$ differ considerably from direct extensional probabilities $P_E(A \circ B)$. Qualitatively, it should be possible to achieve a substantial portion of conjunction ‘fallacies’, even if one uses situations that have previously been shown to reduce CFs (cf. introduction). Quantitatively, BL for instance predicts ‘*differential sample size effects*’ for very low frequencies, which have been confirmed previously

(v. Sydow, 2007). Secondly, it predicts no substantial sample size effect if we are *not* concerned with very low frequencies. Thirdly, BL predicts double CFs and single CFs even with fully transparent tasks, ratings response formats, frequency information, and other strict test conditions. Fourthly, it predicts pattern-sensitivity. The latter three predictions will be addressed by our study (cf. Fig. 1).

Ratings Response Format and Double Conjunction Fallacies

The study presented here has several goals. Firstly, we qualitatively investigate the context conditions under which one may obtain CFs, by simultaneously using transparent tasks, no distractor hypotheses, natural frequencies, clear set inclusions, a clear logical formulation, and a ranking response format. Although other qualitative theories of the CF would predict that CFs should disappear, BL predicts that we should obtain at least a substantial portion of CFs, since we additionally used cues favoring an overall (and intuitive) evaluation of a whole situation. Our tasks clearly go beyond previous tasks concerned with 100 Lindas, which still used a qualitative story and provided no information on particular observations. We used no story at all, but instead provide full frequency information in a contingency table. To combine all these extensional cues goes beyond even some impressive recent documentations of CFs (Sides et al., 2002, Lagnado & Shanks, 2002, Sloman et al., 2003; von Sydow, 2007), that either used no ranking, no explicit frequency information, or more complex and less transparent tasks.

Secondly, the study investigates whether one can obtain double CFs under the mentioned strict conditions. BL predicts *double CFs* ($P(A) < P(A \wedge B) > P(B)$, or $P(B) \leq P(A \wedge B) \geq P(A)$ for $P_E(A \wedge non-B) > 0 \wedge P_E(non-A \wedge B) > 0$) and single CFs, depending on the situation. Previous CF tasks with full frequency information did not address double CFs (Lagnado & Shanks, 2002, cf. v. Sydow, 2007). Corresponding to the definition of representativeness, most work anyway concentrated on single CFs, like $P(\text{bank teller}) < P(\text{feminist} \wedge \text{bank teller})$. In transparent traditional Linda tasks – without fully specified frequency information –, double CFs occurred seldom and only Yates and Carlson (1986) explicitly predicted them (but cf. Thüring & Jungermann, 1990, for problems of this approach).

Thirdly, the sample size is varied and, in contrast to studies concerned with very small set sizes (cf. v. Sydow 2007), BL now predicts no substantial sample size effects.

Fourthly, according to Bayesian logic, subjects should be sensitive to changes of frequency patterns internal and external to the logical class in question, even if the overall number of confirmatory and disconfirmatory cases of that class remains constant. For instance, if we compare the probability ratings of the B hypotheses in the Linda and Johanna school, BL predicts differences, although their frequency and their extensional probability is kept constant.

Method 108 participants from the University of Göttingen voluntarily took part in the study. Participants had to provide probability judgments on rating scales for three schools, like the Linda school. Samples of the graduates of each school were presented in contingency tables with

labeled marginals ‘bank tellers’, ‘no bank tellers’ etc. Table 2 lists the shown frequencies for each school. We used three sample sizes ($N = 36, 75, 96$) with analogous frequency patterns for a Linda school (AND prediction), a Maria school (here A prediction) and a Johanna school (here B prediction). Participants were randomly assigned to one of six combinations of presented schools (always including three different schools and three different sample sizes to prevent a simultaneous occurrence of identical numbers).

Table 2: Frequencies of Observed Cases.

Sample size	School, prediction	$A \wedge B$	$A \wedge \neg B$	$\neg A \wedge B$	$\neg A \wedge \neg B$	Sum
Small sample size	Linda, AND	18	5	6	7	36
	Maria, A	18	15	1	2	36
	Johanna, B	13	5	11	7	36
Medium sample size	Linda, AND	39	11	12	13	75
	Maria, A	39	31	2	3	75
	Johanna, B	26	11	25	13	75
Large sample size	Linda, AND	49	14	16	17	96
	Maria, A	49	40	3	4	96
	Johanna, B	33	14	32	17	96

Participants were explicitly told that they were “not concerned with single probabilities but an *overall evaluation of the situation*”. This was done to outbalance the abundant extensional cues. The rating instruction was (translated): “Please indicate for the different schools how probable you think the hypotheses are.” Participants should use different ratings for the three scales of a school and should proceed intuitively. We use 11 point rating scales without any numbers and only with the labels “low” and “high” probability. The hypothesis read: “ $H1$: “Girls from the Linda [Maria, etc.] school generally become *bank tellers*, regardless of whether they are feminists or not”; “ $H2$: [...] become *active feminists*, whether they are bank tellers or not”; “ $H3$: [...] become *bank tellers and at the same time feminists*”.

Results Figure 1 shows the average probability ratings and mean errors for the three hypotheses for each of the three schools and three sample sizes. As expected, the highest estimated probability for each school corresponded to the highest average ratings and the different sample sizes (low, medium, and high) did not strongly affect the selections. Additionally, Figure 1 shows the predictions of BL and extensional probability. Due to the used rating scale without absolute numbers (and a possible tendency to the middle and effects of a relative instead of an absolute use of scales anyway), both model predictions were linearly transformed by a regression model to provide maximum likelihood estimates on this scale given the models and the data means.

The correlation between BL and data in all schools and sample sizes was $> .98$. In contrast, the correlation between extensional probability and data was highly negative in all Linda schools. The minor deviations from the predictions of BL in the Maria and Johanna condition all went in a direction coherent with BL if subjects would overemphasis small differences in pattern probabilities. This may be

triggered by the instruction that participants should use different ratings within a school.

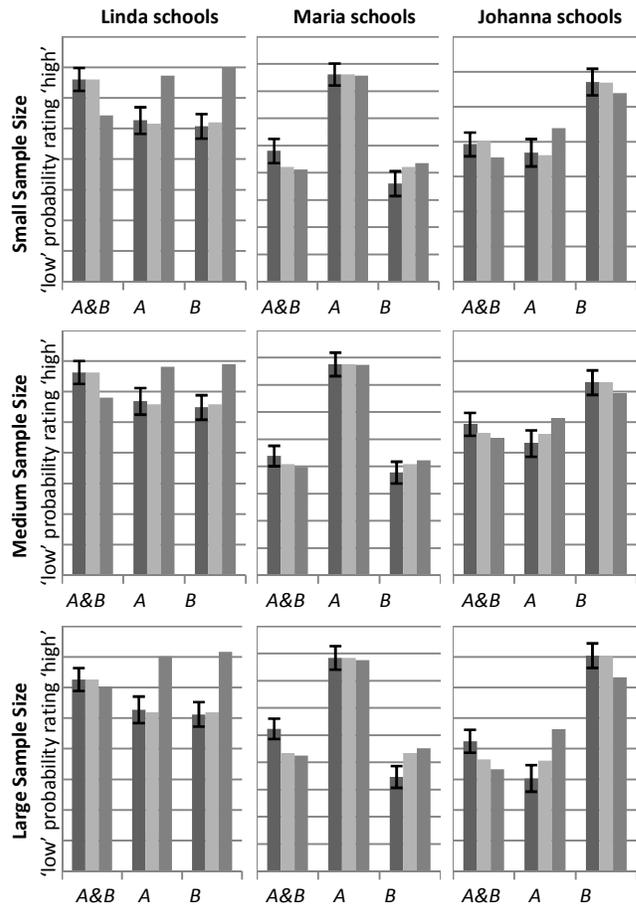


Figure 1: (a) Observed frequencies and (b) mean probability ratings (dark bars), results of the data models for BL (bright middle bars) and extensional probability (right bar) for the three schools, the three hypotheses, and three sample sizes. (Rating scale had 11 steps, without actual numbers, each line representing a step, error bars indicate standard errors.)

With regard to pattern sensitivity, both expected differences are clearly significant for all sample sizes (cf. error bars). Whereas the difference for $P_{emp}(A \wedge B)$ between the Linda and Maria schools may be explained by an effect of the rating scale alone (cf. rescaled extensional probability judgments given the data), this is not the case for the difference of $P_{emp}(B)$ between all Johanna and Linda schools.

Table 3 provides a more detailed level of analysis in showing the overall number and percentage of individual rank order patterns. Although these results were not perfect, the Linda school still lead to 41 % strict and 50 % weak double CFs, despite using very strict conditions. Only 13 % of the qualitative rank orders in the Linda school were coherent with standard extensional probability.

There were significantly more strict double CFs in the Linda School than in the Maria School (15 %) or the Johanna School (19 %) (Linda vs. Johanna: $\chi^2(1, n = 215) =$

Table 3: Percentage (Number) of Rank Order Patterns.

Data Pattern	Linda Conditions	Maria Conditions	Johanna Conditions
1. $c > b > f$	14 % (16)	6 % (6)	5 % (5)
2. $c > b = f$	13 % (14)	3 % (3)	6 % (6)
3. $c > f > b$	13 % (14)	6 % (6)	8 % (9)
4. $b > c > f$	10 % (11)	33 % (36)	5 % (5)
5. $b > c = f$	3 % (3)	19 % (20)	0 % (0)
6. $b > f > c$	6 % (7)	18 % (19) *	4 % (4)
7. $f > c > b$	8 % (9)	1 % (1)	28 % (30)
8. $f > c = b$	4 % (4)	1 % (1)	9 % (10)
9. $f > b > c$	13 % (14) *	5 % (5)	26 % (28) *
10. $c = b = f$	1 % (1)	0 % (0)	5 % (5)
11. $c = b > f$	7 % (8)	8 % (9)	1 % (1)
12. $c = f > b$	1 % (1)	0 % (0)	1 % (1)
13. $b = f > c$	6 % (6)	1 % (1)	0 % (0)

Note: $c := P(b \& f)$, $b := P(\text{bank teller})$, $f := P(\text{feminist})$.
 The cells predicted by Bayesian logic are darkened.
 '*' marks cells predicted by extensional probability.

12.50, $p < .001$). In Table 3 it is shown that the direction of single CFs is reversed in the Johanna and Maria school ($\chi^2(1, n = 205) = 34.45, p < .001$).

Table 4: Percentage (and Number) of Single CFs.

Direction of Single CFs	Linda School	Johanna School	Maria School
$P(A \wedge B) \geq P(B)$	62 % (68)	75 % (80)	31 % (32)
$P(A \wedge B) \geq P(A)$	61 % (67)	25 % (26)	63 % (67)

Note: The predicted cells are darkened.

General Discussion

The findings provide further evidence in favor of Bayesian logic, a novel model testing of dyadic logical hypotheses. Firstly, the results document that CFs can be elicited without a narrative characterizing Linda, only based on quantitative information in contingency tables. This was the case although we simultaneously used natural frequency information (Gigerenzer, 1991, 1998; Hertwig & Gigerenzer, 1999; Zhu & Gigerenzer, 2006), clear set inclusion (Sloman et al., 2003; cf. Johnson-Laird et al. 1999; Barbey & Sloman, in press), a rating response format (Tversky & Kahneman, 1983; Hertwig & Chase, 1998; Lagnado & Shanks, 2002), and highly transparent tasks without distractors (Mellers et al., 2001; cf. Lagnado et al, 2006). These results go beyond other recent impressive documentations of CFs (Sides et al., 2002; Lagnado et al., 2002; Sloman et al., 2003; v. Sydow, 2007). Gigerenzer once wrote: „What is called the ‘Conjunction Fallacy’ is a violation of some subjective theories of probability, including Bayesian theory. It is not, however, a violation of the major view of probability, the frequentist conception.” (1991, p. 92). By contrast, it was shown here that ‘CFs’ are not limited to subjective probabilities and that they need not to be fallacies at all, even when using frequency information. Secondly, even under these strict conditions we

obtained the CFs and a substantial portion of double CFs. Thirdly, although very low sample sizes were shown to have a predicted differential effect on specific hypotheses (v. Sydow, 2007), it was confirmed here that larger sample sizes have no substantial effect. The results support the prediction of pattern-sensitivity.

Although previous quantitative accounts of the CF can explain aspects of our data, none can explain the overall evidence. Firstly, *standard* Bayesian reasoning based on extensional subsets, does not allow for any ‘rational CFs’ (Gigerenzer, 1991; Fisk, 1996).

Secondly, confusing the posterior probabilities, $P(A \wedge B | L)$, with the reverse probabilities, $P(L | A \wedge B)$ (Wolford 1991; Fisk, 1996; cf. Hertwig & Chase, 1998) cannot explain the findings, since they would predict much higher ratings for large sample sizes (relative to lower other sample sizes) and the same probability ratings for the *B* hypothesis in the Linda and the Johanna school (a closer analysis showed that this cannot be explained by presentation order effects).

Thirdly, averaging or signed summation (cf. Yates & Carlson, 1986) cannot be directly applied our tasks, since we provide direct evidence for $P_E(A \wedge B)$. However, a related idea would be to interpret $P(A)_{AV} = (P_E(A \wedge B) + P_E(A \wedge \neg B)) / 2$. This may explain double CFs in the Linda condition, but would predict them in other conditions as well.

Fourthly, the *additional* degree of probability provided by a given data set (‘support’ or ‘confirmation’) is another interesting candidate for explaining traditional CFs (Sides et al., 2002; cf. Lagnado & Shanks, 2002; Bonini et al., 2004). It is assumed that subjects substitute the target measure $P(B \wedge F | D)$ with some confirmation measure, like $P(B \wedge F | D) / P(B \wedge F)$ or $P(B \wedge F | D) - P(B \wedge F)$. Although not tested previously, this account may in principle even predict double CFs. However, it does not explain pattern sensitivity, since two sets with the same extension should not affect support in a different ways. Moreover, support would predict order effects that were neither found here nor in v. Sydow (2007).

Finally, the explanation of ‘representativeness’ has been criticized by Gigerenzer (1996) as a “one-word explanation” that is “vague, undefined, and unspecified” (592, cf. Wolford, 1991, 417). Kahneman and Frederick (2002, 73, cf. 2005) specified representativeness, distinguishing two sub-processes: “(1) a prototype (a representative exemplar) is used to represent categories (e.g. bank teller)”, “(2) the probability that the individual belongs to a category is judged by the degree to which the individual resembles (is representative of) the category stereotype.” BL seems to inherit some aspects from representativeness: BL distinguishes two kinds of processes, a mathematical conscious one, and an intuitive one referring to pattern probability. Moreover, pattern probability may be interpreted as a kind of similarity measure. However, BL differs in its actual predictions from representativeness, particularly as specified by Kahneman and Frederick (2002, 2005). BL regards pattern probabilities and (at least a subset of) ‘CFs’ as potentially rational answers, whereas Tversky and Kahneman (1996) understood them as irrational deviations from extensional reasoning. Moreover, they did not predict CFs under the used strict conditions, with no distractors, clear set inclusion, and frequency information

(Kahneman & Frederick, 2002, 2005). Furthermore, they only predicted single CFs and no double CFs. However, one may extend representativeness to conjunctions if they are treated as prototypic. But in this case representativeness would either be underdetermined or would falsely predict double CFs in all conditions (here the conjunction was always most frequent). Finally, the representativeness heuristic cannot account for pattern-sensitivity (and for differential set size effects, cf. v. Sydow, 2007). Hence, representativeness in its current formulation, cannot explain our data. Nonetheless, I leave it to the reader whether BL should be rather seen as a novel rational formalization of representativeness or an alternative mathematical model.

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