Is there a Monadic as well as a Dyadic Bayesian Logic? Two Logics Explaining Conjunction ‘Fallacies’

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Abstract

Formal logic and probability theory are often considered the most fundamental norms of rational thought, but their application to psychological tasks has raised serious doubts about human rationality. A central finding is that people sometimes judge the probability of a conjunction to be higher than that of its conjuncts (conjunction fallacies, CFs). Bayesian logic (BL, von Sydow, 2011) formalizes subjective probabilities of noisy-logical explanatory patterns (pattern probabilities) instead of extensional probabilities (relative frequencies), and predicts a system of rational inclusion fallacies. This paper distinguishes a monadic from a dyadic pattern explanation of CFs; it tests two corresponding formalizations of BL (the former concerned with cells, the latter with marginals); and it models pattern probabilities in a novel way (based on acceptance thresholds). In an experiment we varied observed frequencies and formulations. The results deviate radically from narrow norms but they corroborate the idea of monadic and dyadic pattern probabilities.

Keywords: Probability judgment; bias; conjunction fallacy; inclusion fallacy; inductive Bayesian logics; predication.

Standard propositional logic and standard probability theory are often considered to be the most fundamental norms of rational thought. Formalizing what is rational has been the subject of study over many centuries. Over 2000 years ago, Aristotle founded formal logic (*Analytica Priora* and *Posteriora*) and basic ideas of probabilistic reasoning (*Topica*). Since logical positivism, the basic calculi of formal logics (Frege, Wittgenstein, Russell, Whitehead) and probability theory (Kolmogorov) have been rigidly formalized in their present form. Yet the standards of rational thought are still being elaborated. Formal epistemology, logics, and mathematics have continued to develop non-standard systems of logic and probability, as either alternatives or refinements to these calculi (e.g., many-valued logics, non-monotonic logics, default logics, and belief functions).

Psychology, however, has often been focused solely on these two basic calculi. This may have contributed to the view that human judgment is essentially biased. In the well known Linda task, people make probability judgments about a person *X* who is described to be a feminist, often *P*(*X* is bank teller & feminist) > *P*(*X* is feminist). This has been called a ‘conjunction fallacy,’ as the probability of a smaller set (the conjunction) cannot exceed that of the larger (one of the conjuncts, Kahneman & Tversky, 1982, cf. 1996). In my view, Gigerenzer (1996) correctly criticized Kahneman and Tversky’s bias-and-heuristic approach, not only because the suggested heuristics were underspecified but also because content-blind application of the “narrow norms” of logics and probability were misguided in the first place. However, Kahneman and Tversky’s (1996) warnings against normative agnosticism seem not without foundation as well.

It is proposed here that in different situations either a monadic or a dyadic Bayesian inductive logic can explain a class of conjunction fallacies (CFs). It is generally suggested, independent of details of the proposed Bayesian models, that there may be monadic CFs based on a probabilistic ‘monadic’ interpretation of two conjuncts. That is, a statement like “*X*’s are young and *Xs* are female” is presumably judged as true if *X*’s are typically young and *X*’s are typically female, even if individual *X*’s are usually not both young and female (monadic interpretation involving a conjunction of two separate monadic propositions). In contrast, “*Xs* are young women”, or “*Xs* are people who are young and female” seems to refer to the traditional interpretation of a conjunction as intersection—that is, *Xs* are individually usually both young and female (dyadic interpretation). Although deterministically a dyadic interpretation concerned with two marginal probabilities, and a dyadic interpretation concerned with logical joint probability distribution (logical patterns) coincide, they can differ in a probabilistic context. Generally, we pursue a third way between a simple application of narrow norms and normative agnosticism by advocating a domain-specific rational approach linked to Anderson’s rational analysis and inspired by the Bayesian renaissance in higher cognition (M. Oaksford, N. Chater, J. Tenenbaum) (cf. von Sydow, 2011).

Truth Table Logics and Probability Theory – Two Narrow Norms for Predication?

The Narrow Norm of Formal Logic If concerned with the truth of sentences involving logical connectives between predicates, such as AND, OR, EITHER OR or NEITHER NOR, standard logical truth-table definitions provide a truth criterion applicable even to contingent sentences: “people from the Linda school become bank tellers AND feminists” or “ravens are black AND they can fly.” Yet interpreting such sentences as $\forall(x)\ B(x) \wedge F(x)$ presents two problems. First, the *problem of exceptions* – predication often involves exceptions (e.g., white albino ravens). Almost none of the assertive predications uttered would be justified, based on a ‘deterministic’ logical truth criterion. A significant suggestion to solve this is instead to use a high probability criterion (cf. Schurz, 2005), where the probability of an empirically justified proposition surpasses a given threshold φ. Second, the *problem of interpretation* is that normal language connectives (e.g., ‘AND’) do not always correspond to analogous ideal language connectives (e.g., the logical conjunction). Some conjunction fallacies may indeed be attributed to misinterpreted connectives (cf. Hilton, 1995, Hertwig, Benz, & Kraus, 2008). Hertwig et al. (2008) have shown that a normal AND may refer to the logical OR (disjunction) or IF THEN (conditional). Beside distinguishing normal from ideal language, our distinction of monadic and dyadic logic suggests two kinds of logical representation.

The Narrow Norm of Extensional Probability If the logical adequacy criterion for predication is replaced by a high probability criterion (using standard probabilities), other problems remain (von Sydow, 2011). The *problem of sample size*: One observation or one thousand observations of $A(x_i) \wedge B(x_i)$ (without exceptions) lead to the same extensional probabilities (relative frequencies) of "*X* are *A* and *B*". This seems inappropriate if our probability is to describe the belief in a hypothesis. (We use probabilities of probability patterns instead.) The *problem of inclusion*: The sentence "ravens are black or they can fly or both" describes a larger set than the conjunction. Thus its higher extensional probability taken as truth criterion implies that we should not prefer predicating the conjunction. The same holds for the tautology "Ravens are black or not, and they can fly or cannot fly," with $P_E(B(r) \text{ T } F(r)) = 1$. It would follow *a priori* that one could never prefer any logical hypothesis over the tautology, irrespective of its meaning. Extensional probability thus does not seem a reasonable adequacy criterion for data-based predication, as it is not sensitive to the data.

Pattern Probabilities and Dyadic Bayesian Logic Von Sydow (2011) has argued that to keep a high probability criterion of predication (Schurz, 2005; Foley, 2009) we have to supplement extensional probability (with its own fields of application) with a kind of subjective pattern probability. Von Sydow has presented a model called Bayesian pattern logic (BL). It is based on Kolmogrov's axioms but allows for conjunction fallacies. It formalizes second-order probabilities of *alternative* logical hypotheses taken as alternative explanations of a situation. The logical hypotheses *l* are ideal probabilistic truth tables (probability tables, PTs) with different degrees of noise *r* (the model can only be sketched here). The 2×2 PTs are possible logical explanations of the data *D*. One first calculates the likelihood $P(D|PT_{l,r})$ and (based on a prior $P(PT_{l,r})$) the posterior probability, $P(PT_{l,r} \mid D)$ using Bayes' theorem. To calculate the final pattern probability of a connective *l* one adds up (over all levels of *r*) the posteriors of all *PTs* corresponding to *l*. – The model predicts a system of frequency-based inclusion fallacies (e.g., conjunction fallacies). Several predictions concerning probability judgments about logical predications have been corroborated: pattern sensitivity; a system of inclusion fallacies; sample-size effects; and trial-by-trial applicability (von Sydow, 2011, von Sydow & Fiedler, 2012).

In this paper we continue testing this model with several frequencies and several hypotheses, but also investigate the (supplementary) model of monadic BL.

The Idea of a Monadic Pattern Logic

Idea of Basic Monadic Pattern Logic Also in propositional logic, monadic logical connectives relate not to two atomic propositions (as do standard dyadic connectives), but to one. Propositional logic contains four monadic connectives: (monadic) affirmation '*A*' (*A*: *T*, Non-*A*: *F*), (monadic) negation (*A*: *F*, Non-*A*: *T*), tautology (*A*: *T*, Non-*A*: *T*) and contradiction (*A*: *F*, Non-*A*: *F*). Again a (monadic) affirmation (e.g., *x* are *A*) cannot have a higher (extensional) probability than the tautology (*x* are *A* or also non-*A*). Hence one may apply the same argument that led to dyadic BL to monadic connectives (particularly the *problem of inclusion*). Since the tautology always has the highest probability, extensional probabilities do not provide a suitable data-based criterion for adequate predication. A pattern probability seems to provide such a criterion. Imagine a pub where most visitors are men (very few women). Thus the sentence "This pub is (generally) visited by men" (affirmation *A*) may have a higher pattern probability than "This pub is visited by people of both genders" (tautology). The monadic model should again provide a (frequency-based) similarity-function between the data and the best explanation. In contrast to dyadic attributes, *monadic* predications are only based on marginals and ignore other attributes (with corresponding joint probability distribution).

Applying Basic Monadic Logic to Dyadic Relations Basic *monadic* pattern probabilities may again be used in *dyadic* relations. In "The pub is visited by people (*X*) who are both male (*M*) AND young (*Y*)," AND seems to refer to the standard logical conjunction (the intersection). For such general (but potentially probabilistic) *dyadic* logical relations we use a standard notation: '$(x)M(x) \wedge Y(x)$', or briefly '$M \wedge Y$'. Yet in the sentence "The pub is visited by people who are male and it is visited by people who are young," the concern may not be the intersection of *M* and *Y*, but rather a combination of two monadic statements, each concerning marginal cases only (notation: '$(x)M(x) \wedge (x)Y(x)$', or briefly '$(M) \wedge (Y)$'). Extensionally, $P_E((M) \vee (Y)) = P_E((M) \wedge (Y))$, but the pattern probability $P_P((M) \wedge (Y))$ may differ from $P_P(M \wedge Y)$ as well as from $P_P((M \vee Y)$. The experiment shows that a conjunction of monadic propositions may correspond to different dyadic connectives. We call the probability of logically *combined* (basic) monadic pattern probabilities simply 'monadic pattern probabilities' as well.

A New Kind of Model A third innovation concerns a new way to determine pattern probabilities, different from von Sydow, 2011. The new formalism is based on establishing subjective degrees of belief and then varying acceptance levels rather than noise levels. This may increase coherence with logical approaches based on acceptance intervals (cf. Schurz, 2005; Foley, 2009), while adding a subjective belief and providing a solution to the problem of inclusion.

A New Model of Monadic Bayesian Logic

This model concerns frequency-based monadic predications based on dichotomous data. Based on individual observations x_i an entity or group of entities *X* (e.g., a group of people, such as "the guests at pub *X*") can be said to be (generally) "*A*" (affirmation), "non-*A*" (negation) or "*A* or non-*A*" (tautology). A specific observation x_i is either *A* or Non-*A* (e.g., a specific visitor of a pub is either male or female). $f(A(x))$, $f(\text{non-}A(x))$) is the data input. The model calculates a posterior probability that combinations of monadic hypotheses are valid given the data and priors.

1. Posterior Distribution of Generative Probabilities We assume that x_i being *A* or non-*A* is a Bernoulli trial, produced by an unchanging generative probability *p*. This generative probability can be differentiated from the observed extensional probability (relative frequency) (and also from the resulting pattern probability of a monadic connective, to be modeled by integrals over acceptance regions for *p*). Given an assumed value of *p* the Binomial distribution then provides the likelihood of the data $P(D|p)$, the number of *k* successes (e.g., $A(x_i)$) in *n* trials:

$$B(k|p,n) = \binom{n}{k} p^k (1-p)^{n-k}$$

Applied to all *p* values, we obtain a likelihood density function (cf. middle Figure 1), which is the kernel of the Beta distribution where only a normalizing constant factor is added. Here the unknown generative probability *p* is the unknown parameter with α-1 = $f(x = A)$ and β-1 = $f(x = \neg A)$:

$$Beta(\alpha,\beta) = P(p|\alpha,\beta) = \text{const.} \cdot p^{\alpha-1}(1-p)^{\beta-1}$$

Taking another Beta distribution as a prior for *p* (we assume flat priors, Beta(1,1)) easily allows to calculate a Beta posterior distribution for *p* (Figure 1).

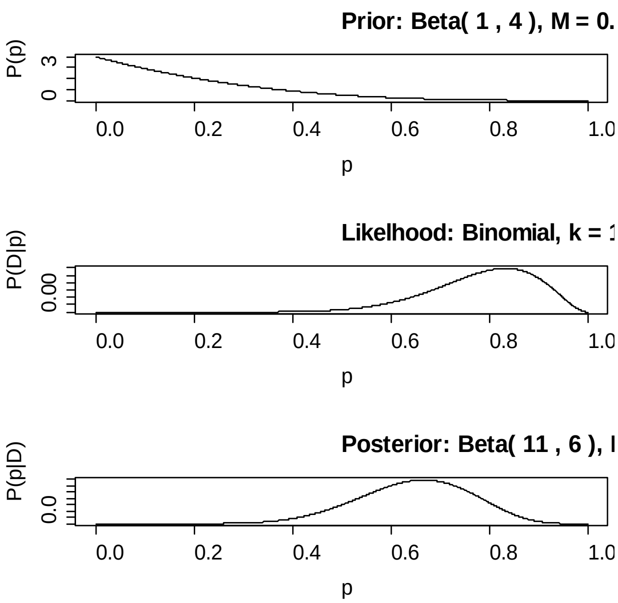


Figure 1: Example for the prior for *p*, the Binomial likelihood and the Beta posterior distribution over *p*.

This provides a subjective posterior probability distribution for the generative probability *p* of the occurrence of an event that is sensitive to sample sizes. The mean of this distribution may be interpreted as a rational point estimate for the probability of the event *A* (or, analogously if sampling was conditional on another event *B*, as $P_{sub}(A|B)$).

2. Pattern Probabilities Based on Acceptance Intervals We build on the idea that predications apply as long as the probability of a sentence is above a threshold (Schurz, 2005; Foley, 2009), although this does not solve the problem of inclusion. We start with ideal generative probabilities (affirmation: p = 1; negation: p = 0; tautology: p = .5) (cf. von Sydow, 2011). We then vary for each monadic hypothesis, H_M, the acceptance threshold *r* and its resulting acceptance interval over *p*. For r = .2 the closed interval for accepting affirmation *A* would be [.8, 1]; for the negation [0, .2]; and for the tautology [.4, .6]. We then calculate for all *r* and each H_M the integral from the lower (r_1) to the higher endpoint (r_2) of these intervals:

$$\int_{r_1}^{r_2} Posterior\ distribution(p, H_M)$$

This provides the subjective probability that for a given *r* the probability *p* of H_M is within an acceptance interval. We additionally relativized the result by the size of the interval (otherwise large *r* trivially would have a high value). Treating the hypotheses as alternatives, we normalize the outcomes. The probability of each monadic hypothesis is finally determined by adding up the results over the different levels of *r*. Figure 2 shows an example where the preference depends on *r* (for r = .3 Pp(A) > Pp(*A* or Non-*A*)).

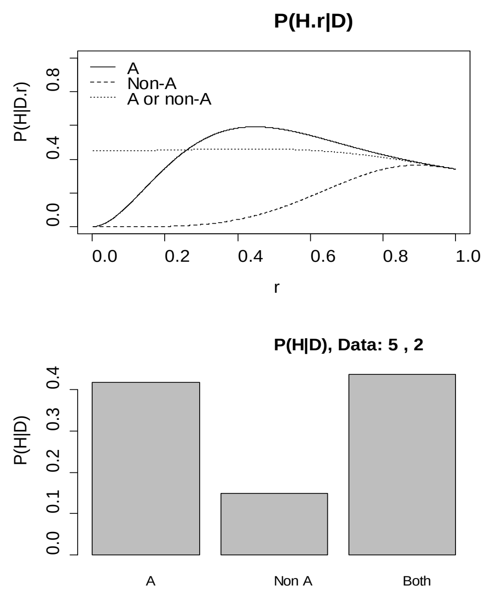


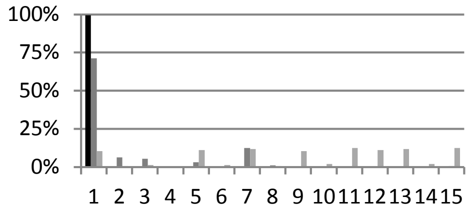
Figure 2: Pattern prob. for diff. levels of *r* (upper) and after integrating over *r* (lower). ($f(A)$ = 5, $f(\neg A)$ = 2, flat priors).

3. Logical Combinations of Monadic Probabilities Based on these monadic pattern probabilities in the narrow sense, we now calculate the conjunctive combination of two monadic connectives. Since monadic attributes by definition should be judged independently, we use the product rule: $P_P((A) \wedge (B)) = P_{PM}(A) \cdot P_{PM}(B)$; P_P((both *A or Non-A*) $\wedge$ (Non-*B*)) = $P_{PM}(\text{both}_A) \cdot P_{PM}(\text{Non-}B)$, $P_P((\text{both}_A) \wedge (\text{both}_B))$ = $P_{PM}(\text{both}_A) \cdot P_{PM}(\text{both}_B)$ etc. The dyadic combination of three monadic propositions leads to 9 possible combinations including a ‘tautology’ (e.g., visitors are male or female, and young or old; i.e., everything is possible).

Data	**Predictions**	**Results**
Observed contingencies	■ Dyadic ■ Monadic ■ Extensional Probabilities of the diff. Hyp. 1 to 15 (cf. Table on the right: dyadic, monadic BL)	Logical hypotheses *H* that are most or second most frequently selected to be most probable given the data and their relative frequency of (*C1* to *C5*)

Panel A

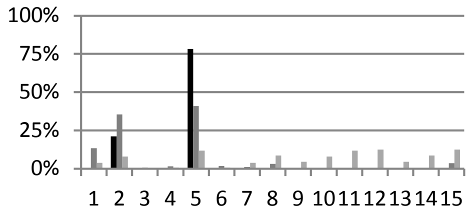
	B	¬B	
A	16	1	27
¬A	2	0	2
	16	1	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	1	1	1	1	1	1	1
choise	*P*(*H*)	99%	71%	77%	73%	41%	68%	91%
Second	*H*	7	7	7	7	7	7	7
choice	*P*(*H*)	1%	12%	18%	18%	36%	27%	9 %

Panel B

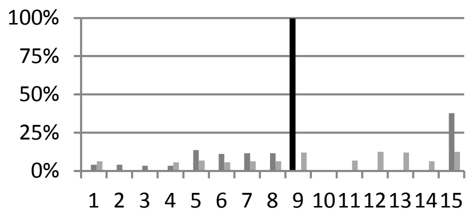
	B	¬B	
A	6	12	18
¬A	0	1	1
	6	13	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	5	5 (2)	5	5	5	5	2
choise	*P*(*H*)	78%	41%	68%	73%	82%	50%	77%
Second	*H*	2	2	2	2	2	2	5
choice	*P*(*H*)	21%	36%	18%	14%	9%	41%	23%

Panel C

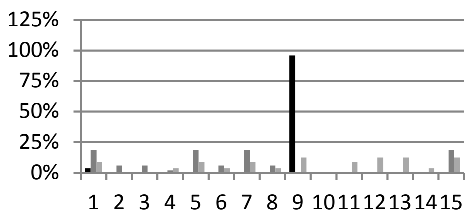
	B	¬B	
A	13	1	14
¬A	0	12	12
	13	13	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	9	15	9	9	15	15	15
choise	*P*(*H*)	99%	38%	95%	95%	27%	64%	95%
Second	*H*	12-	5	12	12	2	2	5
choice	*P*(*H*)	1%-	14%	5%	5%	18%	14%	5%

Panel D

	B	¬B	
A	12	0	12
¬A	0	5	5
	12	5	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	9	1,15*	9	9	1	1	1
choise	*P*(*H*)	96%	18%	59 %	50 %	45%	50%	77%
Second	*H*	1	1,15*	1	1	15	15	15
choice	*P*(*H*)	4%	18%	41%	32%	30%	36%	23%

*1, 15, 5, 7 Cf. main text

Figure 3: Examples for observed contingencies, predictions of dyadic BL, monadic BL and extensional probabilities; and the main results in all 5 conditions (cf. main text for details and Figure 4 for some more results).

Experiment

To test dyadic BL as well as a monadic BL, we investigated which of several alternative sentences involving logical predictions people held to be most probable, given data in a summary format (excluding memory effects). The dyadic and monadic models imply partly different systems of inclusion fallacies. Our design involved 5 conditions involving different formulations (between-subject) and 30 frequency patterns (within-subject scenarios).

The 30 partly critical frequency patterns (scenarios) permit detailed testing of several aspects of the models. This also constitutes a first test of similarities and differences in the predictions. The patterns were however originally designed to test dyadic BL against simpler strategies. The five conditions examine (1) whether one obtains similar results with different formulations, involving different usages of AND (a conjunctive vs. a sum meaning), even if the same model applies; and (2) whether one can use formulations that elicit either a monadic or a dyadic interpretation of the hypotheses used. Two conditions served to elicit answers coherent with dyadic BL (von Sydow, 2011) (dyadic conditions) and three conditions were to elicit answers in line with monadic BL (monadic conditions). Here we aimed to elicit either a purely dyadic or a purely monadic understanding. We used 15 hypotheses in the dyadic condition (perhaps a dyadic cue), and 9 in the monadic condition (Monadic BL can only model these hypotheses). We used formulations in line with either a monadic or a dyadic interpretation. Future work may help to demarcate the role of different cues promoting either monadic or a dyadic understanding of connectives.

Participants One hundred and eight students of the University of Heidelberg participated, in return for course-credit or a recompense of 3 € for 20 minutes.

Method Each scenario concerned the probability of different sentences about a pub *X*. The pubnames were randomly permutated. People always saw a 2×2 contingency matrix with frequency information about visitors being male or female, young or old. In all conditions the task was: "Which sentence would you regard to be most probably valid? Please answer intuitively."

The tested hypotheses corresponded to all 16 possible logical connectives (apart from the contradiction): H1 $A \wedge B$; H2 $A \wedge \neg B$; H3 $\neg A \wedge B$; H4 $\neg A \wedge \neg B$; H5 A; 6. $\neg A$; H7 B; H8 $\neg B$; H9 $A \Leftrightarrow$B; H10 A><B (either or); H11 $A \vee B$; H12 $A \vee \neg B$; H13 $\neg A \vee B$; H14 $\neg A \vee \neg B$; H15 A T B (tautology).

Formulation of the hypotheses in the five conditions (The hypotheses not mentioned were constructed analogously.) *Dyadic Condition 2 (C2), with conjunctive use of "AND".* All sentences read: "Pub *X* is visited by guests who are…" ("In die Kneipe *X* gehen…") and then continued variously—H1: *young and female*; H2: *young and male*; H4: old and male; H5: *young*; H9: either *female and young* or *male and old*; H11: *young and female*, *young and male*, or *old and female*; H15: *young and female*, *young and male*, *old and female*, or *old and male* (all combinations).

Dyadic Condition C1, with noun-adjective combinations and a sum use of "AND": H1: *young women*, etc.; H5: *young people* (men and women); H9: *young women* and *old men*, etc.; H11: *young women, young men*, and *old men*; H15: *young women*, *young men*, *old men* and *old women*.

Monadic Condition C3 (in other respects resembling C1): "The guests of this pub are…" H1: *teenagers and women*; H3: *adults and men*; H5: *teenagers* (woman or also men); H9: (later coded as H15) teenagers or also adults, women or also men; and H16: undecided.

Monadic conditions C4/C5 (with sentences starting like C2): "X is visited by guests who are, but then used a second "who"— H1: guests who are *young* and who are *old*; H5: *young*; H9 (coded H15): are *young or old* and who are *female or male*.

Whereas the contingency table in C1 to C3 gave explicit information about joint frequencies alone, in C4 marginal frequencies were added and in C5 the contingency matrix did *not* provide the joint frequency distribution, but instead marginals only. Since the marginals are implied by cell information, and the cell information is not implied by the marginals, here the use of dyadic BL makes less sense. The frequency information in the tables had labelled cells and marginals. For each of the 30 frequency patterns we used random rotations as a counterbalancing factor.

Results Space limits us here to presenting a selection of the 30 scenarios, to illustrate the strengths and weaknesses of the models. To the left of each panel of Figure 4 is the contingency information provided. Column 2 presents (unfitted) model predictions for monadic BL, dyadic BL and normalized extensional probability (the latter often implies H15 answers). The tables in Column 3 first reiterate which are the most and second most predicted hypotheses and report the corresponding pattern probabilities. Then they show the results, the most and second most frequently selected hypotheses and their relative frequencies.

Panel A of Figure 3 show frequencies which are – from the perspective of dyadic as well as monadic BL – AND patterns with exceptions. As predicted, participants in all conditions often selected the conjunction (H1) (or a corresponding rotation; answers are always re-coded). H11 or H15 (predicted by extensional probability) was rarely chosen, resulting in many double conjunction 'fallacies'.

Panel B shows the same phenomenon for the affirmation (H5). Although monadic and dyadic BL mainly predict H5, monadic BL favors H2 almost equally. In correspondence, the results show at least in C4 and C5 increased H2 choices.

Panel C shows a predicted shift from a (probabilistic) dyadic EITHER-OR (H9) to the monadic tautology (H15).

Panel D shows a different predicted shift form a dyadic either-or (H9) (selected despite great difference in the relative frequencies of the non-empty cells) to an increasing proportion of H1 selections in the monadic conditions. The preference for H1 (over other hypotheses with equally predicted subjective probability) suggests that participants admitted more exceptions than is modelled by our flat prior for *r* (cf. upper part of Figure 3). We have not modelled this yet, but other patterns appear to support this idea.

In Panel E (Figure 4) participants as predicted shifted from a dyadic or-hypothesis (H11) to the monadic tautology (H15). Interestingly, Panel F shows that people (as predicted) can also shift from H11 to H1.

In Panels G and H, the contingencies refer to the same relative frequencies with differing sample sizes. In Panel G the results for monadic and dyadic conditions, mostly as predicted, tended to be undecided between H15 and H5. Although Panel H then shows too high a number of selections of the narrower hypothesis in the clearest monadic condition C5 (H1 cf. Panel D) it corroborates that H15 in all conditions disappears as the dominant hypothesis (sample size sensitivity, cf. von Sydow, 2011).

General Discussion

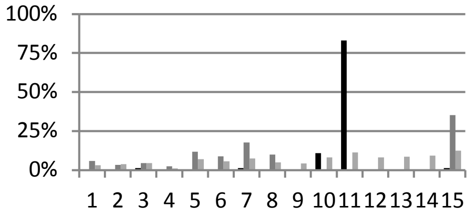
The experiment provides some first evidence for generally advocating two kinds of pattern probabilities involving two systems of logical inclusion 'fallacies' and specifically dyadic and monadic BL. Overall the results showed a very good fit for dyadic BL, ruling out many possible simpler heuristics which may plausibly mimic pattern probabilities (e.g., Panel B and D exclude the dyadic strategy of finding the largest difference between four cells—applicable in Panel C). Likewise, sample-size effects (Panels G and H) appear to exclude strategies using probabilities as input.

There were some substantial deviations from monadic BL. However, the less clear results in C3 than in C5 may be explicable by a lower number of additional monadic cues (e.g., no explicit marginal frequencies). Moreover, we suggested resolving further apparent deviations by modelling a prior for *r* levels. Based on such considerations this first test generally appears to support monadic BL as well.

Without modelling the many theories of the conjunction fallacy (CF) here, the results seem inexplicable by them. First, many are not formulated generally enough to account for all logical connectives. Second, the results rule out theories not concerned with patterns, and those not sensitive to sample size. Finally, I am unaware of any account distinguishing monadic and dyadic predictions. Although for instance the misinterpretation hypothesis (Hilton, 1995), the

Panel E

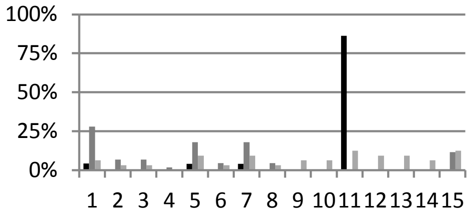
	B	¬B	
A	16	20	36
¬A	23	6	29
	39	26	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	11	15	11	11	15	15	15
choise	*P*(*H*)	83%	35%	73%	55%	41%	68%	45%
Second	*H*	10	7	15	15	1	7	1
choice	*P*(*H*)	11%	18%	27%	27%	32%	27%	41%

Panel F

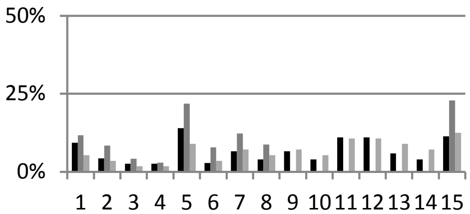
	B	¬B	
A	12	6	18
¬A	6	0	6
	18	6	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	11	1	11	11	1	1	1
choise	*P*(*H*)	86%	28%	59%	32%	36%	50%	86%
Second	*H*	1, 5, 7	5, 7	5	1, 5	5	7	15
choice	*P*(*H*)	04%	18%	14%	18%	27%	23%	9%

Panel G

	B	¬B	
A	3	2	5
¬A	1	1	2
	4	3	

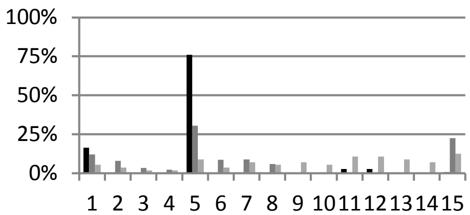


		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	5(15*)	15(5)	15	15	5	5	15
choise	*P*(*H*)	14%	23%	64%	55%	50%	50%	50%
Second	*H*	15	5(15)	5	5	15	15	5
choice	*P*(*H*)	11%	22%	36%	32%	45%	45%	32%

*Cf. main text

Panel H

	B	¬B	
A	27	18	45
¬A	9	9	18
	36	27	



		Dyad.	Mona.	C1	C2	C3	C4	C5
First	*H*	5	5	5	5	5	5	1
choise	*P*(*H*)	76%	31%	68%	68%	64%	73%	50%
Second	*H*	1	15	15	15	15	1	5
choice	*P*(*H*)	16%	23%	27%	27%	23%	18%	45%

Figure 4: Continuation of Figure 3.

unclear-set hypothesis (Sloman et al., 2003), or the confirmation hypothesis (Tentori et al., 2013) may have their domains of application (cf. von Sydow, 2011), it is implausible that they can account for the present results.[1]

This work sketches a novel approach deserving future examination by modelling other variants, parameter-settings and alternative approaches, and refined empirical tests. Using probability as a criterion for rational predication led to intensional (pattern) probabilities, and then to the monadic-dyadic distinction of pattern probabilities. A more content-sensitive model of probability opens the 'rational tool box' of probability measures. Given the multifunctionality of language, we hope this is not a Pandora's box.

Acknowledgments

This work was supported by the grant Sy 111/2-1 from the Deutsche Forschungsgemeinschaft (DFG) as part of the priority program *New Frameworks of Rationality* (SPP 1516). I would like to thank anonymous reviewers for helpful comments and suggestions.

[1] Tentori et al. (2013) show that confirmation can play a role in producing CFs. Due to the rotation of the data confirmation theory would here have to assume either changed relations to the prior expectations and hence less focused selections, or a flat prior for all cells. But in the latter case confirmation, if applicable at all, seems to be highest for cells with highest observed frequencies, falsely predicting selections of H1, H2, H3, or H4 only. In contrast, a *pattern-based* confirmation theory of CFs (suggested in von Sydow, 2011) may well be a candidate worth future exploration.

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